Reference for fact from last time: 4/4/19

- Knop, '96 JAMS, "Automorphisms, root systems, compactifications" Lemma 6.6

G-isotypic decomp of $k[X] = \bigoplus_\lambda k[X]_\lambda$

$$\mathcal{M}' = \{ \alpha \in \Lambda_G \mid \exists\, \lambda, \mu \text{ s.t. } k[X]_\lambda \cdot k[X]_\mu \cap k[X]_{\lambda+\mu-\alpha} \neq \emptyset \subset k[X] \}$$

Then $\mathcal{V} = \{ v \in \check{\Lambda}_G^{\mathbb{Q}} \mid v(\mathcal{M}') \leq 0 \}$

Knop's geometric construction of W_X (overview)

$X = H\backslash G$ homogeneous quasi-affine (hence smooth)

Moment map $T^*X \longrightarrow \mathfrak{g}^* \longrightarrow \mathfrak{g}^*/\!/G = \mathfrak{t}^*/W$

$$T^*X = \mathfrak{h}^\perp \overset{H}{\times} G = \{ x \in X,\ \xi \in \mathfrak{g}^* \mid \xi \in \mathfrak{g}_x^\perp \}$$

$$\begin{tikzcd} T^*X \arrow[d] & \\ \mathfrak{t}^*/W & \mathfrak{a}_X^* \arrow[l] \end{tikzcd}$$

$\mathfrak{a}_X^* \hookrightarrow \mathfrak{t}^*$

Image of $\mathfrak{a}_X^*$ is $\mathfrak{a}_X^*/W(\mathfrak{a}_X^*) \subset \mathfrak{t}^*/W$

$W(\mathfrak{a}_X^*) = N_W(\mathfrak{a}_X^*)/C_W(\mathfrak{a}_X^*)$ ← subquotient of W

Knop's section:

Fix Borel B, $x_0 \in$ open B-orbit of X.

$$\begin{tikzcd} & T^*X \arrow[d] \\ \mathfrak{a}_X^* \arrow[ur, "s"] \arrow[r] & \mathfrak{t}^*/W \end{tikzcd}$$

$\mathfrak{a}_X^* = \Lambda_X \otimes k$

For $\chi \in \Lambda_X$, let $f_\chi \in k(X)^{(B)}$ eigenfunction

$s(\chi) := d_{x_0} \log f_\chi \in T^*_{x_0} X$

extend linearly $\otimes k$

Fact $G \cdot s(\mathfrak{a}_X^*) \subset T^*X$ is dense

$\Rightarrow$ Image of $T^*X \longrightarrow \mathfrak{t}^*/W$ factors through $\mathfrak{a}_X^*/W(\mathfrak{a}_X^*)$

$$\begin{tikzcd} T^*X \arrow[r] \arrow[dr] & \mathfrak{t}^*/W \\ & \mathfrak{a}_X^*/W(\mathfrak{a}_X^*) \arrow[u, hook] \end{tikzcd}$$

By Galois theory,

<u>Defn</u> The normalization of

$$\begin{tikzcd} T^*X \arrow[r, "\text{irreducible fibers}"] \arrow[d] & \mathfrak{a}_X^*/W_X \arrow[dl, dashed] \\ \mathfrak{a}_X^*/W(\mathfrak{a}_X^*) & \end{tikzcd}$$

must be given by some $\underline{W_X} \subset W(\mathfrak{a}_X^*)$

↑ little Weyl gp

<u>Remark</u> Here W_X is constructed as subquotient of W.

There is in fact a canonical embedding $W_X \hookrightarrow W$.

<u>Lemma</u> $k[T^*X]^G \cong k[\mathfrak{a}_X^*]^{W_X}$

i.e., $T^*X /\!/ G \cong \mathfrak{a}_X^*/W_X$